FURYE USULI YORDAMIDA BOSHQARUV MASALASINI YECHISH

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Anotatsiya: Ushbu maqola issiqlik almashish jarayonlarini boshqarish masalalarini yechishda differensial operatorlarning spektrial nazaryasi usullarini qo'llashga bag'ishlangan.

Qaralayotgan masalani yechishda differensial operatorlarning spektral nazaryasi usullaridan ya'ni furye usulidan keng foydalanilgan.Bunda differensial operatorning xos sonlari va shu xos sonlarga mos xos funksiyalari xossalari hamda Grin funksiyasi bilan bog'liq masalalar o'rganilgan.Shuningdek, chegarada boshqaruv funksiyasi topilgan.

Kalit so'zlar: Parabolik tipdagi tenglama,Integral tenglama, Furye usuli, boshlang'ich shart, chegaraviy shart.

Anotation: This article is devoted to the application of the methods of the spectral theory of differential operators in solving the problems of controlling heat exchange processes.

The methods of the spectral theory of differential operators, i.e., the Fourier method, were widely used to solve the problem. In this, the eigenvalues of the differential operator and the properties of the eigenfunctions corresponding to these eigenvalues, as well as the problems related to the Green's function, were studied. Also, the control function was found at the limit.

Key words: Parabolic type equation, Integral equation, Fourier method, initial condition, boundary condition.

Аннотация: Данная статья посвящена применению методов спектральной теории дифференциальных операторов при решении задач управления процессами теплообмена.

Для решения задачи широко использовались методы спектральной теории дифференциальных операторов, т. е. метод Фурье, при этом определялись собственные значения дифференциального оператора и свойства собственных функций, соответствующие этим собственным значениям, а также исследованы задачи, связанные с функцией Грина, а также найдена функция управления на пределе.

Ключевые слова: уравнение параболического типа, интегральное уравнение, метод Фурье, начальное условие, граничное условие.

Hozirgi vaqtda fan texnikaning rivojlanishi ko'plab jarayonlarni nafaqat o'rganish, balki boshqarish imkoniyatlarini ham bermoqda. Juda ko'plab fizik jarayonlarni tadqiq qilish differensial tenglama va matematik fizika masalarini o‘rganishga keladi. Bunga sodda misol qilib ma’lum hududning haroratini aniq temperaturada saqlash masalasini keltirish mumkin.

$u(x,t)$ funksiya quyidag sohada $D=\{(x,t):\ 0<t>T, 0<x<l\}$ issiqlik tenglamasini,

$$u_t=\frac{\partial}{dx}\left[k(x)\frac{\partial u}{dx}\right] \quad (1)$$

boshlang’ich shartni

$$u(x,0)=\phi(x) \quad (2)$$

va chegaraviy shartlarni

$$u(0,t)=\mu(t) \qquad u(l,t)=0 \qquad 0<t<T \quad (3)$$

qanoatlantirsin.

Bu yerda $k(x)$- $k(x)\in C^1[0,l]$, $k(x)>k_0>0$ sterjen materiali tarkibini xarakterlovchi funksiya, $\mu(t)$- $\mu\in C^1[0,+\infty]$ sterjen chetida issiqlikni boshqarish funksiyasi, $\varphi(x)$ - $\varphi\in C[0,l]$ sterjenning boshlang‘ich holati.

Sterjenning o‘rtacha tempraturasi

$$\frac{1}{l}\int_0^l u(x,t)dx=B(t) \quad (4)$$

kabi belgilaymiz. (4) shart sterjenning vaqt mobaynida o‘rtacha temperaturasini berilgan holatda ushlab turishni ko‘rsatadi

Masalaning qo‘yilishi. $B(t)$ funksiya berilgan bo‘lsin. U holda shunday $\mu(t)$ boshqaruv funksiyasini topish kerakki (1)-(3) masala yagona yechimga ega bo‘lsin va bu yechim (4) shartni qanoatlantirsin.

Teorema. Berilgan funksiyalarimiz quyidagi sinfga tegishli bo‘lsin . $k(x)\in C^1[0;l]$, $k(x)>k_0>0, \varphi\in C[0,l]$ va $\mu\in C^1[0,+\infty]$.U holda (1)-(4) masalaning yechimi mavjud va yagona.

Isbot.Chegaraviy shartlarni bir jinsli qilish uchun (1)- tenglamaning umumiy yechimini quyidagi ko‘rinishda izlaymiz.

$$u(x,t)=v(x,t)+\omega(x,t) \quad (5)$$

Natijada $v(x,t)$ funksiyaga bog’liq quyidagi masalaga kelamiz.

$$v_t=\frac{\partial}{dx}\left(k(x)\frac{\partial v}{dx}\right)+\frac{\partial}{dx}\left(k(x)\frac{\partial\omega}{dx}\right)-\omega_t. \quad (6)$$

$$v(x,0)=\varphi(x)+\frac{x-l}{l}\mu(0) \quad (7)$$

$$v(0,t)=0, \quad v(l,t)=0 \quad (8)$$

Bu yerda $\omega(x,t)=\frac{l-x}{l}\mu(t)$ ko‘rinishga ega.

(6)-tenglamani yechish uchun differensial operatorlarning spektral nazaryasi usullaridan ya’ni furye usulidan foydalanib yechimini topamiz.

Furye usuliga binoan (6) tenglamaning bir jinsli qismining xususiy yechimlarini

$$v(x,t)=X(x)T(t) \quad (9)$$

ko‘rinishda izlaymiz.

Natijada quyidagi tenglikka ega bo‘lamiz.

$$X(x)T'(t)=k'(x)X'(x)T(t)+k(x)X''(x)T(t)$$

$$\frac{T'(t)}{T(t)}=k'(x)\frac{X'(x)}{X(x)}+k(x)\frac{X''(x)}{X(x)}=-\lambda^2 \quad (10)$$

(10)- tenglikdan xos qiymat masalasiga kelamiz.

$$\left.\begin{array}{l}\frac{\partial}{\partial x}\left[k(x)\frac{\partial}{\partial x}X(x)\right]=-\lambda^2X(x)\\ X(0)=X(l)=0\end{array}\right\}$$

(11)

(11)- masala Shturm-Liuvill masalasi bo’lib, uning xos qiymatlari masalaning xos sonlarini λ_n, xos sonlarga mos xos funksiyalarni $X_n(x)$ deb belgilaymiz.

(6)-tenglamaning umumiy yechimini quyidagi ko‘rinishda qidiramiz.

$$v(x,t)=\sum_{n=1}^{\infty}T_n(t)X_n(x) \quad (12)$$

Quyidagi belgilashni amalga oshiramiz:

$$\int_0^l X_n^2(x)dx=a_n \quad (13)$$

Endi (6) tenglamamizni bir jinsli bo‘lmagan qismini furye koeffitsientlarini topamiz.

Tasdiq 1. Quyidagi

$$\frac{\partial}{dx}\left(k(x)\frac{\partial\omega}{dx}\right)-\omega_t=\frac{-k'(x)}{l}\mu(t)+\frac{x-l}{l}\mu'(t) \quad (14)$$

funksiyaning furye koeffitsienti

$$\frac{\mu(t)}{la_n}\int_0^l -k'(x)X_n(x)dx+\frac{\mu'(t)}{l(-\lambda_k^2)a_n}\left[\int_0^l k'(x)X_n(x)dx+lk(0)X_n'(0)\right] \quad (15)$$

teng .

Isbot. (14) funksiyaning furye koeffisientini topish uchun quyidagi integralni hisoblaymiz.

$$\frac{1}{\int_0^l X_n^2(x)dx}\int_0^l\left(\frac{-k'(x)}{l}\mu(t)+\frac{x-l}{l}\mu'(t)\right)X_n(x)dx=\frac{\mu(t)}{l\int_0^l X_n^2(x)dx}\int_0^l -k'(x)X_n(x)dx+$$

$$+\frac{\mu'(t)}{l\int_0^l X_n^2(x)dx}\int_0^l (x-l)X_n(x)dx=\frac{\mu(t)}{l\int_0^l X_n^2(x)dx}\int_0^l -k'(x)X_n(x)dx+\frac{\mu'(t)}{l(-\lambda_k^2)\int_0^l X_n^2(x)dx}\int_0^l -\lambda_k^2(x-l)X_n(x)dx=$$

$$=\frac{\mu(t)}{l\int_0^l X_n^2(x)dx}\int_0^l -k'(x)X_n(x)dx+\frac{\mu'(t)}{l(-\lambda_k^2)\int_0^l X_n^2(x)dx}\int_0^l (x-l)\frac{\partial}{\partial x}\left[k(x)\frac{\partial}{\partial x}X_n(x)\right]dx=$$

$$=\frac{\mu(t)}{l\int_0^l X_n^2(x)dx}\int_0^l -k'(x)X_n(x)dx+\frac{\mu'(t)}{l(-\lambda_k^2)\int_0^l X_n^2(x)dx}\cdot\left[\int_0^l k'(x)X_n(x)dx+lk(0)X_n'(0)\right].$$

(13)-belgilashdan foydalansak quyidagi ko‘rinishga teng bo‘ladi:

$$\frac{\mu(t)}{la_n}\int_0^l -k'(x)X_n(x)dx+\frac{\mu'(t)}{l(-\lambda_k^2)a_n}\left[\int_0^l k'(x)X_n(x)dx+lk(0)X_n'(0)\right].$$

Tasdiq.2. Quyidagi

$$\varphi(x)+\frac{x-l}{l}\mu(0) \qquad (16)$$

funksiyaning furye koeffitsienti

$$\frac{1}{a_n}\int_0^l \varphi_n X_n(x)dx+\frac{\mu(0)}{l(-\lambda_k^2)a_n}\cdot\left[\int_0^l k'(x)X_n(x)dx+lk(0)X_n'(0)\right]$$

(17)

teng.

Isbot. (16) funksiyaning furye koeffitsientini topish uchun quyidagi integralni hisoblaymiz.

$$\frac{1}{\int_0^l X_n^2(x)dx}\int_0^l\left(\varphi(x)+\frac{x-l}{l}\mu(0)\right)X_n(x)dx=\frac{1}{\int_0^l X_n^2(x)dx}\int_0^l \varphi_n X_n(x)dx+$$

$$+\frac{\mu(0)}{l(-\lambda_k^2)\int_0^l X_n^2(x)dx}\cdot\left[\int_0^l k'(x)X_n(x)dx+lk(0)X_n'(0)\right].\qquad ,$$

(13)-belgilashdan foydalansak quyidagi ko‘rinishga teng bo‘ladi:

$$\frac{1}{a_n}\int_0^l \varphi_n X_n(x)dx+\frac{\mu(0)}{l(-\lambda_k^2)a_n}\cdot\left[\int_0^l k'(x)X_n(x)dx+lk(0)X_n'(0)\right].$$

$$\frac{1}{l}\int_0^l X_n(x)dx=c_n,$$

(6)-(8) masalaning umumiy yechimi

$$v(x,t)=\sum_{n=1}^{\infty}\left(\frac{1}{a_n}\int_0^l \varphi_n X_n(x)dx+\frac{\mu(0)}{l(-\lambda_k^2)a_n}\cdot\left[\int_0^l k'(x)X_n(x)dx+lk(0)X_n'(0)\right]\right)e^{-\lambda_k^2(t-\tau)}X_n(x)+$$

$$+\sum_{k=1}^{\infty}\int_0^t\left(\frac{\mu(\tau)}{la_n}\left(\int_0^l -k'(x)X_n(x)dx\right)+\frac{\mu'(t)}{l(-\lambda_k^2)a_n}\left[\int_0^l k'(x)X_n(x)dx+lk(0)X_n'(0)\right]\right)e^{-\lambda_k^2(t-\tau)}d\tau X_n(x)$$

Demak (1)-(3) masalaning umumiy yechimi quyidagiga teng bo‘ladi.

$$u(x,t)=\sum_{n=1}^{\infty}\frac{e^{-\lambda_k^2 t}}{a_n}\int_0^l \varphi_n X_n(x)dxX_n(x)+\sum_{k=1}^{\infty}\frac{\mu(t)}{l(-\lambda_k^2)a_n}\left[\int_0^l k'(x)X_n(x)dx+lk(0)X_n'(0)\right]+$$

$$+\sum_{k=1}^{\infty}\frac{k(0)X_n'(0)X_n(x)}{a_n}\int_0^t \mu(\tau)e^{-\lambda_k^2(t-\tau)}d\tau-\frac{x-l}{l}\mu(t)$$

Quyidagi tenglikdan

$$\sum_{k=1}^{\infty}\frac{\mu(t)}{l(-\lambda_k^2)a_n}\left[\int_0^l k'(x)X_n(x)dx+lk(0)X_n'(0)\right]=\frac{x-l}{l}\mu(t)$$

umumiy yechimning ko‘rinishi

$$u(x,t)=\sum_{n=1}^{\infty}\frac{e^{-\lambda_k^2 t}}{a_n}\int_0^l \varphi_n X_n(x)dxX_n(x)+\sum_{k=1}^{\infty}\frac{k(0)X_n'(0)X_n(x)}{a_n}\int_0^t \mu(\tau)e^{-\lambda_k^2(t-\tau)}d\tau$$

teng bo‘ladi.

(4)-shartga ko‘ra

$$\int_0^l\left(\sum_{n=1}^{\infty}\frac{e^{-\lambda_k^2 t}}{a_n}\int_0^l \varphi_n X_n(x)dxX_n(x)+\sum_{k=1}^{\infty}\frac{k(0)X_n'(0)X_n(x)}{a_n}\int_0^t \mu(\tau)e^{-\lambda_k^2(t-\tau)}d\tau\right)dx=lb(t)$$

teng.

Integral bilan yig’indini o‘rnini almashtiramiz va quyidagi belgilashni bajaramiz.

$$\int_0^l X_n(x)dx=c_n$$

Natijada quyidagi tenglik hosil bo‘ladi.

$$\sum_{n=1}^{\infty}\frac{e^{-\lambda_k^2 t}c_n}{a_n}\int_0^l \varphi_n X_n(x)dx+\sum_{k=1}^{\infty}\frac{1}{a_n}k(0)X_n'(0)c_n\int_0^t \mu(\tau)e^{-\lambda_k^2(t-\tau)}d\tau=lb(t).$$

Quyidagi almashtirishlarni bajaramiz.

$$\Phi(t)=\sum_{n=1}^{\infty}\frac{e^{-\lambda_k^2 t}c_n}{a_n}\int_0^l \varphi_n X_n(x)dx,$$

$$K(t)=\sum_{k=1}^{\infty}\frac{1}{a_n}k(0)X_n'(0)c_n e^{-\lambda_k^2 t}$$

$$\int_0^t K(t-\tau)\mu(\tau) = lb(t) - \Phi(t) - K(t)$$

Yuqoridagi tenglama 1-tur Volterra tenglamasi bo‘lib,uning yechimi Laplas almashtirish va Millen almashtirish usulidan foydalanib topiladi.Bunday natijani akademik Alimov.SH.O ilmiy ishlarida keltirilgan.

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